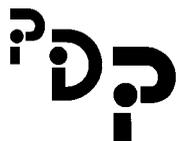


*Life of Fred*<sup>®</sup>  
*Advanced Algebra*  
*Expanded Edition*

Stanley F. Schmidt, Ph.D.



Polka Dot Publishing

## What Is in Advanced Algebra?

All kinds of stuff. It's the second half of algebra. You've seen the first half, and therefore, things like  $2x = 14$  are not very scary anymore. This is the rest of high school algebra. After completing this book, you will have all the algebra you need for college calculus. The only two other math courses needed for calculus will be a geometry course (with an emphasis on doing proofs) and a trig course.

In beginning algebra we've already done most of the classic word problems such as . . .

JENNIFER CAN DIG A DITCH IN 4 HOURS.

JASON CAN DIG IT IN 5 HOURS. IF THEY  
WORK TOGETHER HOW LONG WILL IT TAKE?

OR

JASON RUNS DOWN THE HALL AT 5 MPH.  
WHEN HE'S 50 FEET AWAY, JENNIFER RUNS  
AFTER HIM AT 6 MPH. HOW LONG BEFORE  
THEY'RE HAPPY?

We've already learned 94.7% of factoring. The only thing left is the factoring of  $x^3 + y^3$ , which is  $(x + y)(x^2 - xy + y^2)$  and the factoring of  $x^3 - y^3$ , which is  $(x - y)(x^2 + xy + y^2)$ . Oops! I guess you've just finished factoring.

You've gone through the agony of learning to add algebraic fractions:

$$\frac{x+2}{x+5} + \frac{x+1}{x+4} = \frac{(x+2)(x+4)}{(x+5)(x+4)} + \frac{(x+1)(x+5)}{(x+4)(x+5)} = \frac{(x+2)(x+4) + (x+1)(x+5)}{(x+4)(x+5)}$$

and the terror of the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The further you go in math, the less memorizing and the less computational cookbook stuff you encounter. You will find that *understanding* rather than just being a good tape recorder starts to matter more.

In Fred's everyday life in this book, he runs into things that would baffle a beginning algebra student. For example, in Chapter 3 you learn how to solve  $2^x = 5$ . In Chapter 6 you learn how to battle the dreaded Snow King using a Waddle-Ray which can be obtained at your local doughnut store. In Chapter 9 we add up an infinite number of numbers, such as  $1/2 + 1/6 + 1/18 + 1/54 + \dots$ , and we get an answer! A finite answer. Not your usual old stuff.

We are often asked for the Big Overview: What is ahead and which order to study the subjects.

After learning arithmetic and pre-algebra, the steps are:

<i>Life of Fred: Beginning Algebra</i>	}	taken in this order
<i>Life of Fred: Advanced Algebra</i>		
<i>Life of Fred: Geometry</i>		
<i>Life of Fred: Trigonometry</i>		
<i>Life of Fred: Calculus</i>		
<i>Life of Fred: Statistics</i>		
<i>Life of Fred: Linear Algebra</i>		

(*Statistics* may be taken before *Calculus*.)

And now the scary question . . .

## *Are You Ready for Advanced Algebra?*

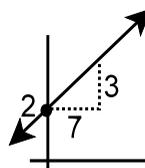
Here are some questions taken from *Life of Fred: Beginning Algebra*. The answers are given on the next two pages. This will give you an indication of whether you are ready for advanced algebra.

B e g i n n i n g

A l g e b r a

Q u i z

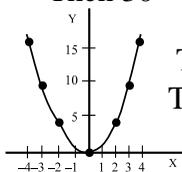
1. If two sets have the same number of elements in them, are they equal? (from Chapter 1)
2. If the diameter of a circle is exactly 4 feet (that would make a nice-sized pizza), what is the exact circumference? (from Chapter 2)
3. We need 30 cc of 25% cough medicine. In the medicine cabinet was a solution that was too weak. It contained only 20% of the cough medicine by volume. Another bottle was too strong. It contained 35% cough medicine by volume. How much of each of these bottles should be mixed together to obtain 30 cc of 25% cough medicine? (Chapter 3)
4. Plot  $y = x^2$ . (Chapter 5)
5.  $(y^{20})^3 = ?$  (Chapter 6)
6. Factor  $6x^2 + 29x + 35$  (Chapter 7)
7. Simplify  $\frac{x^2 - xy + 3x - 3y}{x^2 - 2xy + y^2}$  (Chapter 8)
8. Solve  $\sqrt{2y - 3} + 3 = y$  (Chapter 9)
9. Solve  $5x^2 = -4x + 13$  (Chapter 10)
10. What is the equation of the line whose graph is (Chapter 11)
11. Suppose the domain is  $\{5, 6, 7\}$  and the codomain is all rational numbers. For each element in the domain, pretend it is the radius of a pizza and would be mapped to the area of the pizza (by the formula  $A = \pi r^2$ ). So 5 would be mapped to  $25\pi$ . Is this a function? (Chapter 11)
12. Solve  $48 - 3x > 36$  (Chapter 12)



## Answers to the Beginning Algebra Quiz

1. No. Two sets are equal if they have the same elements in them. For example,  $\{\alpha, \beta\}$  is equal to  $\{\beta, \alpha\}$ . The sets  $\{\#\}$  and  $\{\text{pen}\}$  have the same number of elements in them, but they are not equal.
2.  $4\delta$  feet    The formula relating the diameter of a circle and its circumference is  $C = \delta\delta$ . If  $d = 4$ , then  $C = 4\delta$ .
3. Let  $x$  = the number of cc of the 20% medicine used.  
 Then  $30 - x$  = the number of cc of the 35% medicine used (since we have to make up a total of 30 cc of medicine).  
 Then  $0.20x$  = the amount of cough medicine taken from the 20% bottle.  
 Then  $0.35(30 - x)$  = the amount of cough medicine taken from the 35% bottle.  
 The total amount of medicine needed is 25% of 30 cc, which is 7.5 cc.  
 The equation then is  $0.20x + 0.35(30 - x) = 7.5$   
 Solving, we obtain  $x = 20$  cc of the 20% medicine.  
 Then  $30 - x = 10$  cc of the 35% medicine.

4.



This can be done by point-plotting.

- Three steps:
1. name  $x$  values
  2. find the corresponding  $y$  values
  3. plot those points until you have enough of them to “connect the dots.”

5.  $(y^{20})^3 = y^{60}$  by the rule  $(x^a)^b = x^{ab}$

6.  $6x^2 + 29x + 35 = (3x + 7)(2x + 5)$

7.  $\frac{x^2 - xy + 3x - 3y}{x^2 - 2xy + y^2} = \frac{(x - y)(x + 3)}{(x - y)(x - y)} = \frac{x + 3}{x - y}$

8.  $y = 6$ .                       $\sqrt{2y - 3} + 3 = y$   
     $\sqrt{2y - 3} = y - 3$                       isolating the radical  
     $2y - 3 = y^2 - 6y + 9$                       squaring both sides  
     $y = 6$  OR  $y = 2$                       solving by factoring

$y = 6$  checks in the original problem.

$y = 2$  doesn't check in the original problem.

9. First place  $5x^2 = -4x + 13$  into the form  $ax^2 + bx + c = 0$  and then use the quadratic formula.

$$x = \frac{-4 \pm \sqrt{16 - (4)(5)(-13)}}{10} = \frac{-4 \pm \sqrt{276}}{10}$$

and if you (optionally) simplified  $\sqrt{276} = \sqrt{4} \sqrt{69} = 2\sqrt{69}$ , then your

final answer would be  $x = \frac{-2 \pm \sqrt{69}}{5}$

10.  $y = (3/7)x + 2$  The slope of the line is  $3/7$  and its y-intercept is 2. The slope-intercept form of a line is  $y = mx + b$  where  $m = 3/7$  and  $b = 2$ .

11.  $25\delta$  is not a rational number. Therefore, this is not a function. A function, by definition, maps each element of the domain to exactly one element of the codomain.  $25\delta$  is not in the codomain.

12.  $48 - 3x > 36$

Subtract 48 from both sides  $-3x > -12$

Divide both sides by  $-3$   $x < 4$

(When you multiply or divide an inequality by a negative number, you have to change the sense of the inequality:  $>$  becomes  $<$ .)

small essay

### Being Happy in Math

One important part of success in math is working at the right place in your math education. Right now,  $2 + 2 = 4$  would bore the socks off of you and  $\int_{x=0}^1 \cosh x \, dx$  might just be a little too much. (That last thing is from fourth semester calculus.)

You just took the beginning algebra quiz. Do you want to be happy? It's important you are working at the right place. If you took some wishy-washy beginning algebra course and you got only seven or eight questions right on this quiz, then the smart thing to do would be to grab a copy of *Life of Fred: Beginning Algebra Expanded Edition* and zip through the 104 lessons before starting this book.

end of small essay

## *A Note to Students*

Fred has just received an honorable discharge from the army and is taking the bus home to KITTENS University in Kansas. You are about to join him on that bus ride.

On the two-day ride you will experience all of advanced algebra—everything you will need to know before studying trigonometry and calculus. You will have it all.

The supplies you'll need for the trip:

1. pencil or pen
2. paper
3. a handheld calculator that has the keys: sin, log, !, and  $y^x$ . This is the last calculator that you will ever need. You can usually find them for \$15 or less. Stop! Last week I was in one of those stores that sell everything for a dollar and found one of those calculators for a buck.

You will not need a “graphing calculator.” I don't even own one, and I do a lot of math.

When I studied algebra, my teacher told the class that we could reasonably expect to spend thirty minutes per page to master the material in the old algebra book we used. With the book you are holding, you will need two reading speeds: slower when you're learning algebra and faster when you're enjoying the life adventures of Fred.

Throughout the book are sections called *Your Turn to Play* and *Cities*, which are opportunities for you to interact with the material. Just reading the problems and reading my solutions doesn't work. You have to do them. Education *does* take effort.

Our story begins at noon on Monday and ends on Tuesday evening. Each lesson is a day's work. After 10 chapters you will have mastered all of advanced algebra.

Just before the Index, the **A.R.T.** section begins. **A.R.T.** = All Reorganized Together. This section very briefly summarizes advanced algebra. If you have to review for a final exam or want to quickly look up some topic ten years after you've read this book, the **A.R.T.** section is the place to go.

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## Chapter One

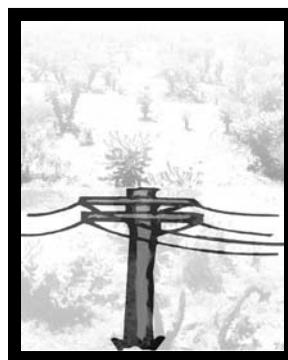
### Lesson One—Ratios, Median Averages, Proportions



Fred looked out the bus window. The cold, white Texas landscape might have seemed bleak to many people, but to him it was a joy. He was heading north—back to his home in Kansas.

He thought about the last four days. Friday had been his sixth birthday. So much had happened since then: his “abduction” into the army, all the new friends he had met, his hurt rib, and his honorable discharge.\*

Now he could look out at the passing telephone poles and just imagine them as a clock ticking away the hours till he reached his office at KITTENS University (Kansas Institute for Teaching Technology, Engineering and Natural Sciences), where he has lived for the last five years.



view from the window

It would be good to get out of his hospital nightshirt with the little blue and green frogs all over it. Tomorrow would be Tuesday and maybe by then his rib wouldn’t hurt so badly. With a good night’s sleep and a fresh bunch of clothes, he’d meet his 8 a.m. class.

The telephone poles whooshed by, one after another. He looked out of the bus window and unconsciously began to count them: five poles passed for every three beats of his heart. (He could feel his pulse as little stings in his hurt rib.) The **ratio** of the passing poles to the heartbeats was 5:3. Ratio means division, so 5:3 could also be written as  $5 \div 3$ .

His eyes began to close, shutting out the snowy scene. A little nap would help pass the time. Five-thirds would become ten-sixths would become fifteen-ninths. He’d soon be asleep.

“Hey! How old are you?”

Fred was startled by the half-shouted question. He received a little poke in the ribs and then he was fully awake.

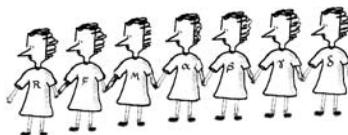
“I said how old are you,” the little girl repeated.

“I’m six. I just turned six last Friday.”

---

\*This story is told in *Life of Fred: Beginning Algebra*

She said, “Oh” and ran to her friends in the back of the bus. They were all about four years old and were all dressed identically in gray-brown dresses. They giggled and chattered.



Fred might easily have been mistaken for a four-year-old. He had always been less than the **median** weight for his age. (The median weight means that half the people are heavier and half the people are lighter than that weight. The median is one of the three kinds of averages studied in beginning statistics.) Fred, at 37 pounds, was definitely less than the median weight for his age. Maybe only 4% of boys his age weighed less than he did.

He noticed that the ratio of the telephone poles that the bus was passing to his pulse was now 5:4. His heart was beating more quickly. Getting awakened with a question and a poke would cause most people’s hearts to beat faster.

Oh well he thought, and after a few moments he began to drift back to sleep.

He could hear her coming. Some little girl running up the aisle to his seat. It was a different girl than the first one. Instinctively, he put his arms around his ribs to protect them against further assault.

“What’s your name?” she blurted. She had been sent on a mission to find this out.

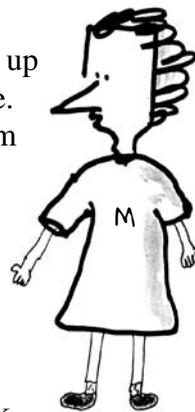
Fred, who had read all the James Bond books, thought of answering Gauss, Fred Gauss but instead he simply said, “Fred.”

“Oh” was her only response, and she ran to the back of the bus to report her findings to her girlfriends.

Fred was so used to being around the students at KITTENS that these four-year-olds seemed to him to be so . . . he couldn’t think of the word. They seemed to be so *immature*.

A woman, also wearing one of the gray-brown uniform dresses, came up to Fred and smiled. “Hi. My name is Cheryl Mittens. I hope my little girls haven’t been bothering you.”

“Not too much. Could you tell me what’s going on? Are they just playing or something?”



“Well, you might call it that,” Cheryl said. “They’re working on earning a badge for their uniforms. It’s the Getting-to-Know-People badge. The first requirement is to learn to make contact with some fellow that they want to get to know.”

“Oh,” said Fred. (He was starting to sound like the girls.) “But I’m 50% older than these girls.” (He had done the math in his head:  $4 \times 1.50 = 6$ .) Fred thought to himself *It’s a little early for those girls to be thinking about finding a husband.* “I read in a marriage manual that the prospective bride should be at least 90% of the age of the groom. That would make the ratio of their ages 9:10. Right now, the ratio of their ages to mine is 4:6.”

Cheryl laughed. “I guess you’re right. The girls aren’t thinking about marriage right now. And don’t worry. They’ll never be 90% of your age.”

Fred’s heart raced at the thought of marriage to one of those *children*. Maybe he’d be lucky and he’d be a hundred years old before they were grown up enough to be of the appropriate age. Then he wondered how long it really would be before they were 90% of his age—before the ratio of their ages to his age was 9:10. He let  $x$  equal the number of years from now until that happened. In  $x$  years, the girls would be  $4 + x$  years old and he would be  $6 + x$  years old. In  $x$  years, the ratio

$4 + x : 6 + x$  would be the same as 9:10.

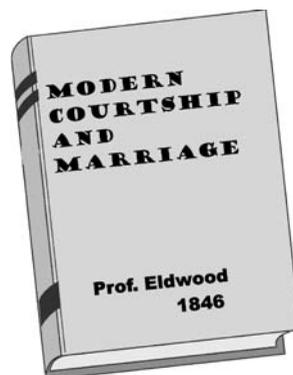
$$\frac{4 + x}{6 + x} = \frac{9}{10}$$

Two ratios set equal to each other is called a **proportion**. When we solved fractional equations in beginning algebra, we found an expression that all the denominators would evenly divide into and multiplied every term by that expression. In this case  $10(6 + x)$  will do the trick:

$$\frac{(4 + x)10(6 + x)}{6 + x} = \frac{9 \cdot 10(6 + x)}{10}$$

The denominators disappear

$$(4 + x)10 = 9(6 + x)$$



Be warned: This book might be a little out-of-date.

Distributive property

$$40 + 10x = 54 + 9x$$

the distributive property

$$a(b + c) = ab + ac$$

Subtract  $9x$  from each side

$$40 + x = 54$$

Subtract 40 from each side

$$x = 14$$

So in 14 years (which would make the girls 18 and Fred 20), they would be 90% of his age.

Gasps! Fred thought to himself. That's way too soon. He wanted to wait until he was at least 50 before he'd have to think about such things.

He needed to change the subject. "These girls are too young to be Girl Scouts, but they're wearing some kind of uniform. Are they part of some group?" he asked.

Mrs. Mittens replied, "They're even too young for Blue Birds. My three daughters, Fredrika, Meddie, and Rita, and all their girlfriends in the neighborhood, have made up a little club and I'm their club leader. We call ourselves the Dust Bunnies. That's why our uniforms are grayish-brown."

small essay

### Life

Life is painful. The only choice you have is how you want to take the pain.

You can take the pain now. It will be short and sharp. And it will end.

You can avoid the pain now. It will come later. It will be a dull pain that will last and last.

Taking out a piece of paper and writing your answers is pain now. Not learning the math well now and getting "lost" later in the book is pain later.

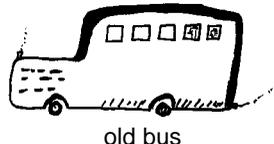
end of small essay

Please take out a sheet of paper and write your answers to the *Your Turn to Play* on the next page before you look at my answers.

You will learn a lot more than if you just read the question and read the answer.

*Your Turn to Play*

1. Which is larger: 6:5 or 9:8?
2. In some of the old math books they used to write a proportion as 2:3::6:9. What would the double colon in the middle represent?
3. When Fred first counted the ratio of passing telephone poles to his heartbeats, he found it was 5:3. Suppose the driver of the bus increased his speed. What might the new ratio look like?
4. As Fred was counting the ratio of passing telephone poles to his heartbeats, suppose (Heaven forbid!) his heart stopped beating.
  - ✓ The bus driver wouldn't like this because he would have to stop the bus and do some heart surgery or something.
  - ✓ The readers of the advanced algebra book wouldn't like it because the book would end too soon.
  - ✓ Mathematicians wouldn't like it because the resulting ratio is 5:0. Why would they object?
5. Solve  $\frac{x+3}{x+13} = \frac{3}{5}$
6. The bus driver is 25 years old. The bus is 35 years old. How long will it be before the driver is 75% of the age of the bus?
7. What is the median average of:  
5, 8, 9, 9, 10, 14, 18, 19, 19?



..... **COMPLETE SOLUTIONS** .....

- 6:5 means  $6 \div 5$  which is 1.2.  
9:8 means  $9 \div 8$  which is 1.125.      6:5 is larger.
- A proportion is the equality of two ratios.  
The expression  $2:3::6:9$  would translate into  $2:3 = 6:9$  or  $\frac{2}{3} = \frac{6}{9}$
- Instead of 5:3 it might be 6:3 or 7:3. Any answer you gave which was in the form  $x:3$  where  $x > 5$  would have been fine.
- A ratio of 5:0 means  $\frac{5}{0}$  which is division by zero. Mathematicians don't especially like that. It is similar to going up to someone and saying, "The snamplefork is overzipped." Division by zero *doesn't have any meaning*. When you divide 2 into 6 you get an answer of 3.

$$\begin{array}{r} 3 \\ 2 \overline{)6} \end{array}$$

You check your answer by multiplying 2 by 3 and hoping to get 6.

If you try to divide by zero,  $0 \overline{)6}^?$  what could the answer be? What number could you replace the question mark with so that the answer would check? Suppose the answer were 97426398799426.

Suppose 
$$\begin{array}{r} 97426398799426 \\ 0 \overline{)6} \end{array}$$

This answer wouldn't check since  $0 \times 97426398799426 \neq 6$ .

- $$\frac{x+3}{x+13} = \frac{3}{5}$$

$$\frac{(x+3)\mathbf{5(x+13)}}{x+13} = \frac{3\mathbf{5(x+13)}}{5}$$

Multiplying both sides by  $\mathbf{5(x+13)}$

$$(x+3)5 = 3(x+13)$$

$$x = 12$$

- Let  $x =$  the years until the bus driver is 75% of the age of the bus.  
Then in  $x$  years, the bus driver will be  $25 + x$  years old.  
Then in  $x$  years, the bus will be  $35 + x$  years old.

$$\frac{25 + x}{35 + x} = 75\%$$

$$\frac{25 + x}{35 + x} = \frac{3}{4}$$

$$\frac{(25 + x)4(35 + x)}{35 + x} = \frac{3(4)(35 + x)}{4} \quad \begin{array}{l} \text{Multiply both sides} \\ \text{by } 4(35 + x) \end{array}$$

$$(25 + x)4 = 3(35 + x)$$

$$100 + 4x = 105 + 3x \quad \text{Distributive property}$$

$$x = 5 \text{ years}$$

7. The median average of 5, 8, 9, 9, 10, 14, 18, 19, 19 is the number in the middle when they are all arranged in order of size. In this case it is 10.

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